Wage Determination, Capital Accumulation and Profit Rate Dynamics under Two Production Scenarios*

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Abstract

Possible grounds for Marx's hypothesis of a tendentially falling rate of profit are investigated using a dynamic model of an accumulating capitalist economy. The core argument concerns the implications for profit dynamics of the process of wage determination and the time path of technical change when the rate of accumulation depends on the magnitude of the profit rate. The paper establishes conditions under which the "steady-state" rate of profit falls over time, but argues that these conditions, while plausible, are at best problematically understood as innate features of capitalism.

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1. Introduction

The analytical foundation of Karl Marx's fatal prognosis for capitalism is his theory of tendentially falling rates of profit. The general contours of this theory are well known. Marx's major premise, established in Chapter 13, Volume III of *Capital*, is that the rate of profit measured in value terms falls as the organic composition of capital rises, holding the rate of surplus value constant. His minor premise, argued in Chapter 25, Volume I of *Capital*, is that the "general law of capital accumulation" implies a tendency to increase the organic composition of capital through technical changes which involve the substitution of machines for direct labor in production.

Marx's derivation of tendentially falling profit rates from these premises rests on crucial subsidiary hypotheses concerning the economic processes of wage determination and capital accumulation.[1] Specifically, Marx sees a direct relationship between the wage rate and aggregate demand for labor power by capitalists, and sees the rate of profit as to some extent *self-regulating* through its impact on the rate of capital accumulation. While the modern debate about Marx's theory has tended to focus on the issue of wage determination, both hypotheses have problematic implications for Marx's account. My purpose in this paper is to address these problems in a unified analytical framework. In doing so, I take up a vision of capitalist dynamics first suggested by David Laibman over 20 years ago.

[1] Marx's conclusion also depends on the assumption, which I will not examine here, of a close correspondence between the rates of profit calculated respectively in value and dollar terms.

Consider first the logic and role of wage determination in Marx's theory. Marx clearly understands the "industrial reserve army" created by the accumulation pattern identified above to have a depressing effect on wage rates.

> Taking them as a whole, the general movements of wages are exclusively regulated by the expansion and contraction of the industrial reserve army, and this in turn corresponds to the periodic alternations of the industrial cycle. They are not therefore determined by the variations of the absolute numbers of the working population, but by the varying proportions in which the working class is divided into an active army and a reserve army....The relative surplus population is therefore the background against which the law of the supply and demand of labour does its work. [I, 790-92]

Since Marx also asserts that the industrial reserve army allows capitalists to maintain or extend the working day (I, 789-90), it follows that the general law of capital accumulation implies an increase in the rate of surplus value concomitant with the rise in organic composition. Thus Marx's Volume III thought experiment in which the rate of surplus value is held constant while allowing organic composition to rise at first glance seems of dubious relevance.

This difficulty is probed from a number of perspectives beginning with Okishio (1961) and continuing through Frank Thompson's recent work (Thompson (1995)). Thompson shows that given a "minimal assumption" about the impact of labor demand on the wage rate which is weaker than the position taken by Marx in the passage cited above, the *ceteris paribus* impact of any capital-using, labor-saving techical change is to *increase* the rate of profit, in direct contrast to Marx's claim.

Consequently, at least on the basis of a comparative static vision of the capitalist economy, a "supply and demand equilibrium" approach to wage determination leads to results which contradict Marx's conclusions. Thus, if the Marxian hypothesis of tendentially falling profit rates is to be maintained in this context, the response of wage rates to technical changes which increase the organic composition of capital must be different than that envisioned by Marx; specifically, the wage rate must be inversely related to capitalists' aggregate demand for labor power. David Laibman (1982) and Duncan Foley (1986) have separately proposed alternative hypotheses which are tantamount to assuming a constant wage *share* of aggregate net product. In a forerunner to this paper (Skillman, 1996), I use a strategic bargaining model of capitalist labor markets to suggest how this outcome might result under competitive conditions.

However, it is possible that the foregoing analysis leads us astray by proceeding from a *static* vision of the capitalist economy. This brings us to the second issue addressed in this paper, which concerns the *dynamic* logic of capital accumulation in Marx's theory of tendentially falling profit rates. To begin his analysis of "the general law of capitalist accumulation," Marx posits that in the absence of biased technical change the profit rate is more or less self-regulating through its effect on the rate of capital accumulation:

> If the quantity of unpaid labour supplied by the working class and accumulated by the capitalist class increases so rapidly that its transformation into capital requires an extraordinary addition of paid labour, then wages rise and, all other circumstances remaining equal, the unpaid labour diminishes in proportion. But as soon as this diminution touches the point at which the surplus labour that nourishes capital is no longer supplied in normal quantity, a reaction sets in: a smaller part of revenue is capitalized, accumulation slows down, and the rising movement of wages comes

up against an obstacle. [I, 771]

The rate of accumulation lessens; *but this means that the primary cause of that lessening itself vanishes*, i.e. the disproportion between capital and exploitable labour-power. *The mechanism of the capitalist production process removes the very obstacles it temporarily creates*. [I, 770; emphasis added]

Passages such as this remind us that for Marx capitalism must be characterized in terms of laws of *motion* rather than comparative static conditions such as those typically studied in the theoretical literature on Marx's macroeconomic theory of capitalist profit. They might also suggest a potential inconsistency in that theory: if threats to capitalists profits are automatically regulated by offsetting variations in the rate of accumulation, as Marx indicates, such that "[t]he mechanism of the capitalist production process removes the very obstacles it temporarily creates," then in what sense can it be said that there is a tendency of the rate of profit to *fall*, as opposed to a tendency of the rate of profit to oscillate around some long-term average?

Following the pioneering efforts of Laibman (1977) and Kliman (1988), this paper investigates the possibility of resolving this conundrum by treating (organic composition-increasing) technical change as *continuous* rather than *episodic*, in the context of a dynamic model of a capitalist economy. The intended contribution of this paper is to be somewhat more transparent about the conditions yielding Marx's hypothesis, and to show how assumptions about the process of wage determination affect the results.

On the first point, the present approach differs from that of Laibman primarily in assuming that individual capitalists seek to maximize *profit* rather than the profit *rate*. The latter assumption, which departs from the standard neoclassical case, is not justified by Laibman, so it is not clear what he is presuming about the logic of capitalist competition. On the other hand, the profit-maximization condition can be shown to emerge from competitive stock market conditions.

The approach developed here diverges from the more recent effort of Kliman in two important ways: first, the rate of capital accumulation is understood as endogenous rather than exogenous, and second, microeconomic relationships are stated in real rather than labor-value terms. The first departure seems more appropriate to Marx's framework given the foregoing comments, while the latter can be justified by noting that capitalists respond to real (or anticipated real) variables rather than value-theoretic categories. In any case, the present approach avoids the need to justify given value-theoretic axioms.

I show that there is a *logically* coherent basis within a dynamic framework for the conclusion of tendentially falling profit rates, understood as a fall in the steady-state rate of profit. However, this conclusion rests on restrictive premises, the relevance of which must ultimately be justified on empirical rather than theoretical grounds. Marx's hypothesis of tendentially falling profit rates is thus seen to be *contingent* rather than *general*.

2. First scenario: fixed-coefficients technology

Model

Consider a capitalist economy with an indeterminate time horizon, where time proceeds by discrete periods indexed by $t = 0, 1, 2,...$. Let there be F identical firms and L_t identical workers at time t. There is one produced good, which is also used as an input to production. The duple (k_t , l_t) denotes the (homogeneous) inputs of the produced good and labor power (assuming a particular solution to the labor extraction problem) respectively used by a representative firm in production.

The production technology is given initially by a fixed-coefficients production function of the form $x_t = \min\left\{ k_t/a_t \, , \, l_t/b_t \right\}$, where ($a_t$, b_t) are positive constants and a_t is less than one for all t. (An alternative variable-proportions specification of the production technology is considered in section 5.) For the produced good to be used as an input in a given period, it must have been produced and saved from an earlier period.

There are markets for the produced good and for labor (power). Normalize the price of the consumption good at $p = 1$, and let (r_t , w_t) denote the profit and wage rates obtaining in period t. All buyers and sellers in the two markets are price-takers. Firms seek to maximize profits given prices and the production technology.

Let (L_t , K_t) represent the aggregate stocks of labor power (the "labor force") and capital available for use in production at time t. If suppliers of labor power are wage-takers, the resulting labor power supply curve is understood to be perfectly wage-elastic at the "subsistence" wage $\underline{w}$ up to L_t, at which point it becomes perfectly inelastic with respect to the wage rate. L_t is assumed to grow at an exogenously given rate g per period. The savings rate out of labor income is assumed to be zero. The capital stock is assumed to grow at the rate $s\ r_t$ from period t, where s is the exogenously given savings rate out of capital income, net of a fixed depreciation rate per period of $d \leq 1$.

Technical changes are represented as rates of change in the production parameters, $(\dot{\alpha}_t, \dot{\beta}_t)$. For example, a capital-using, labor-saving innovation, which increases the organic composition of capital when adopted universally, implies $\dot{\alpha}_t > 0, \dot{\beta}_t < 0$. For convenience, I will generally assume that these rates of change are constant, such that $\dot{\alpha}_t = a, \dot{\beta}_t = b$. Technical innovations become instantaneously and generally available at the beginning of a period, before production decisions are made. Firms are assumed to adopt innovations if and only if they are *viable*, which is to say they are cost-reducing for given wage and depreciation rates.

Optimal production conditions

Assuming that any technical changes occur instantaneously in the passage between periods and that viable innovations are adopted universally, in any given period *t* each firm seeks to minimize costs of production given the production technology $x_t = \min\left\{ k_t / \alpha_t , l_t / \beta_t \right\}$. The cost-minimizing production plan entails $x_t = k_t / \alpha_t = l_t / \beta_t$,

which implies in turn that a representative firm's input demand functions are given by $k_t^d = \alpha_t x_t, l_t^d = \beta_t x_t$, where superscript "*d*" indicates quantity demanded. The corresponding unit cost of production for a representative firm is therefore given by $c^* = \alpha_t d + \beta_t w_t$.

Summing across the *F* indentical firms yields the aggregate input demands

(1a) $$K_t^d = \alpha_t X_t$$

(1b) $$L_t^d = b_t X_t \ ,$$

where $K_t^d = Fk_t^d$, $L_t^d = Fl_t^d$, and $X_t = Fx_t$.

Aggregate feasibility and distribution conditions

Aggregate feasibility of production plans requires that $L_t^d \leq L_t$, $K_t^d \leq K_t$, since firms can't employ more inputs than exist in total. Purely for simplicity the analysis will proceed from an arbitrarily chosen period in which the capital supply constraint is binding (the razor-edge case in which the labor supply constraint is also just binding is allowable but unnecessary to consider separately), so that questions concerning the rationality of positive capital accumulation when capital is in excess supply need not be addressed unless made necessary by the dynamic results to follow. Thus we have

(2) $$K_t = K_t^d$$

as an initial condition for the dynamic analysis.

There are only two classes, capital and labor, between which aggregate net product is distributed exhaustively, or

(3) $$X_t - d\ K_t = w_t L_t^d + r_t K_t \ .$$

Note capitalists are assumed to calculate the rate of profit on the entire capital stock existing at time t.

Wage determination

We arrive now at the first of the two critical issues in the Marxian theory of profit identified in the introduction. Since by assumption capitalists are always on the "short side" of the market (save for the razor-edge case in which both supply constraints are binding), a Walrasian equilibrium analysis implies that the wage is always determined at the minimum possible level consistent with continued supply, i.e. the subsistence wage. As is well known, this manner of wage determination ensures that Okishio result that any viable technical innovation must increase the rate of profit upon general adoption.

Laibman (1982) and Foley (1986) have suggested alternative value-theoretic specifications which in a one-produced-good model are equivalent to the condition that total wages are a constant share of net product. As we will see, this specification violates Thompson's Minimal Assumption (and consequently Marx's explicitly stated vision of wage determination); thus, as Roemer argues in a review of Foley's work (Roemer, 1990), the microeconomic basis for this specification is not at all clear.

In an earlier paper (Skillman (1996)) based on the work of Rubinstein and Wolinsky (1985), I demonstrate that the Laibman-Foley wage condition is a possible outcome of a matching and bargaining process in the limit as market frictions approach zero. Thus I argue that this wage condition can be thought of as consistent with the steady state of a particular dynamic competitive market process. This steady state presumes a constant *rate of flow* of buyers and sellers through the market for labor power, rather than static supply and demand conditions as in the Walrasian model.

Consistent with Rubinstein and Wolinsky's approach (see Osborne and Rubinstein, Ch. 7, pp. 146-7), the wage share constant is assumed to be determined by the relative numbers of capitalists and workers. Thus the wage determination equation is given by

(4) $$w_t L_t^d \big/ (X_t - d\,K_t) = q_t = K_t \big/ K_t + L_t \,.$$ [2]

Expressions (1) - (4) give us five equations in the five endogenous variables, K_t^d, L_t^d, X_t, w_t and r_t. We can therefore completely determine the macroeconomic system described by these equations. Substituting the wage equation (4) and the capital market equilibrium condition (2) into the income distribution equation (3) and rearranging yields
a closed-form expression for the contemporaneous rate of profit:

(5) $$r_t = L_t(1 - d\,a_t) \big/ (K_t + L_t)a_t \,.$$

Note result (5) confirms Laibman and Foley's separately derived conclusion that any capital-using, labor-saving technical change (such that $da_t > 0$) must lower the prevailing rate of profit, other things equal. But this is a comparative static result, and Marx insists in Chapter 25 of Volume I that such technical change is bound up in an intrinsically dynamic process of capital accumulation. Let us therefore now inquire after the laws of motion that might characterize this process.[3] After analyzing this given condition (4), I'll consider a process of wage determination more in keeping with Marx's Chapter 25 story.

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[2] To be strictly in keeping with the Rubinstein and Wolinsky bargaining story, the wage share should be determined by the ratio of *firms* to the sum of firms and workers, since firm owners rather than their units of capital stock engage workers in negotation. However, given identical scale limits across firms, this specification yields an equivalent result to that yielded by the simpler specification given in (4). The more precise specification would be useful if we sought to explore the impact of *centralization* on the dynamics of capital accumulation and profit.

[3] Note further that if q_t is not a function of K_t and L_t, as specified in equation (4), then the result in (5) simplifies to $r_t = (1 - q_t)(1 - d\,a_t) \big/ a_t$, which shows that the rate of profit necessarily falls as capital intensity increases. But then we have ruled out the self-regulating aspect of accumulation Marx emphasizes

Capitalist dynamics and the (un)steady-state rate of profit

Marx's depiction of the dynamic capitalist macroeconomy in Chapter 25, Volume I of *Capital* reflects his rejection of the Malthusian theory of population growth, according to which the latter is increasing in the wage rate, and his insistence on the self-regulating logic of dynamic profit determination. These commitments notwithstanding, Marx concluded that labor will tend to be in excess supply, creating an "industrial reserve army" of the unemployed, and profit rates will tend to fall. Our task is to discover whether these claims are mutually consistent, and if so under what conditions.

Marx's postulates are reflected in the model's assumptions that the growth rate in labor supply is exogenously given, and the growth rate in capital depends on the current rate of profit. Letting dotted variables denote rates of growth, we have $\dot{E}_t = g$ and $\dot{K}_t = s\, r_t$, where s is the exogenously given rate of saving out of capital income, and the rate of capital growth is gross of the fixed rate of depreciation d. Anticipating Marx's "general law of capital accumulation," posit a time trend for the capital intensity variable, $\dot{a}_t \geq 0$.

To consider the impact of these dynamic processes on the prevailing rate of profit, take the total differential of (5). Substituting these growth conditions and rearranging yields

(6) $$\dot{r}_t = q_t (g - s\, r_t) - {d\, a_t \dot{a}_t} \big/ {1 - d\, a_t} .$$

Note that if the technical change trend is not dependent on the profit rate, the wage share, or the growth parameters, this dynamic system is *stable*: "too high" rates of profit imply that the profit

in the first section of Chapter 25 in Volume I, and have implausibly divorced wage determination entirely from relative supplies of capital and labor.

rate will tend to fall, while if profit rates are "too low", they will tend to rise, just as Marx suggests in section 1 of Chapter 25 in Volume I of *Capital*.

In light of this dynamically self-regulating aspect of profit, it seems reasonable to define the *steady state* of this system as that value of the profit rate r_s such that $\dot{r}_t = 0$. Here we encounter the second critical aspect of Marx's theory of profit pinpointed in the introduction. Suppose that we are in the capitalist world Marx describes in section 1 of Chapter 25, before Marx incorporates the hypothesis of a dynamic trend of capital deepening. In this case no consistent time trend is assumed to exist for the capital intensity parameter, and capital-using, labor-saving technical changes are represented as incidental "comparative static" changes, the typical procedure in the modern literature on Marx's theory of profit.

It is easily seen that in this case the steady state rate of profit takes the value

(7) $$r_s = g/s \quad .$$

Since in this case the steady-state rate of profit does not depend on any technical parameters, it is invariant to technical change, and thus there can be no "tendency" for the rate of profit to fall in this dynamic system. As we saw in the previous section, a capital-using innovation will reduce the *prevailing* rate of profit, but the rate of accumulation will then adjust to drive the rate of profit back up to the constant steady-state level.

We thus see the critical importance to Marx's theory of the next step in his Chapter 25 argument. If in contrast to the foregoing we impute a time trend to technical change, for example $\dot{\alpha}_t = a > 0$, the steady-state rate of profit becomes

(8) $$r_s = \frac{g - [\delta\alpha_t a/(1-\delta\alpha_t)\theta_t]}{s} .$$

Since the "steady state" now includes a time-dependent term, it may follow a particular time trend. To determine this, note that given (8) the steady-state growth rate of capital is necessarily slower than the exogenously given growth rate of labor, g . *Consequently, the wage share* q_t *falls across time, and thus so must the steady-state rate of profit*. We might imagine that wages are determined instantaneously at the beginning of each new period, and that the profit rate adjusts immediately to reflect the new production and wage conditions. There is then no *within-period* pressure for further adjustment--thus the "steady state"--until the ensuing change in class proportions necessitates another instantaneous correction in the wage share.

Equation (8) thus establishes a dramatic and perhaps startling result: *despite* the self-regulating aspect of capital accumulation described by Marx in section 1 of Chapter 25, the consequence of the time trend of technical change he analyzes in the remainder of the chapter is persistently to depress the steady-state profit rate, ensuring, as he concluded in Volume III of *Capital*, a "tendency for the rate of profit to fall."

Marx's "canonical case" of wage determination

The preceding result was based on a version of the Laibman-Foley "constant wage share" condition, which is not necessarily the process of wage determination envisioned by Marx in his account of the "general law of capitalist accumulation." To the contrary, this account seems to presume a wage determination process more or less consistent with standard supply and demand theory, understood in the context of an accumulating economy. In this connection, I refer to Marx's "canonical case" as one in which the supply curve is "⌋-shaped" and an industrial reserve army exists, so that at the margin the wage rate is constant at the "subsistence" level, i.e., equation (4) is replaced by

(4′) $w_t = \underline{w}$

for aggregate stocks of labor power in the proximity of L_t (for an arbitrarily given reference period t).

Applying equation (4′) and the optimizing conditions (1) to the aggregate distribution equation (3) and rearranging yields

$$r_t = \frac{1 - d\,a_t - \underline{w}b_t}{a_t}, \tag{9}$$

which given constant returns to scale and wage-taking behavior is equivalent to the expression for individual firms' rate of profit. *Consequently, Marx's hypothesis of a tendentially falling rate of profit is inconsistent with his "canonical case" of wage determination*, even in the context of a continuously accumulating economy.

3. Second scenario: Cobb-Douglas technology

I now consider a neoclassical-style "variable proportions" technology which allows "market clearing" in the standard sense to occur in the market for labor power.

Variable proportions technology

Now suppose that the production technology for all firms is given by a Cobb-Douglas production function of the form $x = k_t^a l_t^{1-a}$, $a \in (0,1)$. Each firm is assumed to have a capital stock $\underline{k}_t$ in period t. In each period each firm correspondingly chooses labor (power) and capital good inputs to maximize profit taking wage and depreciation rates as given, i.e. solves $\underset{l_t, k_t}{Max}\; k_t^{a_t} l_t^{1-a_t} - w_t\, l_t - d\, k_t + l\,(\underline{k}_t - k_t)$. I'll assume the depreciation rate is such that each firm's capital stock constraint is binding, so that $\underline{k}_t = k_t{*}$ for all t.

Within-period outcomes

Standard optimization analysis generates the following labor demand functions for a representative firm,

(9) $$l_t^d = \underline{k}_t \left({(1-a_t)}/{w_t} \right)^{1/a_t},$$

which yields

(10) $$L_t^d = K_t \left({(1-a_t)}/{w_t} \right)^{1/a_t}$$

as the aggregate labor demand equation.

Notice that in this case the contemporaneous wage rate enters the expression for quantity demanded of labor power, and thus now has an allocative role in addition to the distributive one examined above. This is a general consequence of variable proportions technology, and implies that one must first examine the impact of wage changes on optimal labor input use before considering Marx's hypothesis.

The viability condition and the aggregate distribution equation (3) remain the same as in the previous scenario. Therefore we can proceed directly to exploring the alternative wage determination scenarios.

Wage determination

Reversing the order followed in the previous section, let's begin with the "canonical case" arising from a reverse L-shaped supply curve in the presence of a reserve army. In this case equation (4´) holds. Applying this and the labor demand equation (10) to the distribution condition (3) and rearranging yields

(11) $$r_t + d = a_t \left(\frac{(1-a_t)}{\underline{w}} \right)^{(1-a_t)/a_t},$$

which does not depend on relative stocks of labor power and capital, and thus not on the rate of capital accumulation. Given wage-taking behavior and constant returns to scale, however, (11) is also the expression for an individual firm's rate of profit, so that *viable trends technical change must once again increase the rate of profit over time.*

In contrast, suppose the wage share condition (4) holds. In this case the contemporaneous rate of profit does depend on stocks of capital and labor, as in the previous scenario, so that we can proceed as before to calculate the contingent "steady-state" rate of profit. Doing so yields

(12) $$r_s = \frac{g}{s} - \frac{aa_t(K_t + L_t)}{sK_t}.$$

This result is comparable to equation (8). Given (12) the capital stock must grow more slowly than the labor stock over time, and thus the wage share falls. Given any (viable) capital-using, labor saving technical trend, r_s must fall over time.

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