

# **Accumulation, Time Preference, and the Persistence of Exploitation\***

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In *A General Theory of Exploitation and Class* (1988) (and the related popularization *Free to Lose* (1988)), John Roemer poses a fundamental challenge to the methodological and substantive givens of traditional Marxist economic theory. With respect to methodology, Roemer argues that the labor theory of value was at best unnecessary as a vehicle for capturing the key aspects of the Marxist critique of capitalism, in particular the idea that capitalist profit is based on the exploitation of labor. He posits in its place the analytical framework of “rational-choice” Marxist models, which adapted elements of the neoclassical paradigm to incorporate Marxist concerns about the class distribution of power and material well-being. With respect to substance, Roemer challenges the traditional identification of production as the essential locus of capitalist exploitation, arguing that *differential ownership of relatively scarce productive assets* (DORSPA) is necessary and generically sufficient for the existence of capitalist exploitation, with or without direct capitalist control of production.<sup>1</sup>

Not surprisingly, Roemer’s work attracted a barrage of criticism from Marxist social scientists who claimed that his analytical framework was essentially un-Marxian and thus constitutionally incapable of capturing the core insights of Marx’s critique of capitalism. This critical response especially targeted Roemer’s use of the neoclassical competitive market model based on rationally optimizing actors. My purpose in this paper is address a particular aspect of this targeted attack.

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<sup>1</sup>Roemer uses the simple phrase DOPA (*differential ownership of productive assets*) for this phenomenon. My coinage of the added adjective phrase “relatively scarce” requires some explanation. “Scarcity” in the neoclassical sense refers to the incurrence of opportunity costs in supplying commodities for which there is positive demand. Scarcity in the sense required for Marxian does not imply the presence of opportunity costs, although this is not ruled out. Rather what counts is that the total supply of productive assets is limited, *relative to current demand*. If no opportunity costs exist, this implies that profit and interest, thus surplus value, thus capitalist exploitation, corresponds to the transfer of economic rents. The key issue addressed by the present paper is thus the conditions under which relative scarcity of capital goods might be maintained in the face of capital accumulation.

Noting the static nature of Roemer's formal market models, a number of Marxist critiques have argued that his framework is essentially incapable of explaining the *persistence* of DORSPA and thus capitalist exploitation in the presence of ongoing capital accumulation. This point is developed most thoroughly in the first round of critical response by Devine and Dymski (1991), who develop a simple example to illustrate the intuitive point that relative capital asset scarcity can be undone by unchecked accumulation. Roberto Veneziani (2000) has challenged Roemer's argument more directly by extending his models to the multiperiod case and characterizing the robustness of exploitation in a particular multi-period version of a Walrasian framework.

The chief claims of Veneziani's carefully articulated analysis might be summarized as follows: (1) in an explicitly intertemporal context, a distinction must be drawn between "within-period" and "whole-life" exploitation, with the former arguably being the more relevant normative standard; (2) according to the "within-period" definition, persistent DOPA (not DORSPA, note) does not ensure the persistence of exploitation, which tends to disappear in the limit as  $T$  approaches infinity; and (3) again by the "within-period" standard, there is partial convergence to a single, petty-bourgeois class. (These claims are embodied in Veneziani's Proposition 3, p. 27).

Veneziani and other critics are surely correct to insist that exploitation must be understood in the context of an essentially dynamic vision of capitalist competition. I want to argue, however, that their criticisms of Roemer on this point are ill-founded if they are meant to suggest that his analytical framework is fundamentally *inadequate* rather than simply *incomplete* with respect to the treatment of capitalist accumulation and

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exploitation. Essentially, Roemer's critics on this score deny him a method of abstraction by successive approximation that is approvingly attributed to Marx.

My purpose in this paper is to demonstrate that a natural dynamic extension of Roemer's rational-choice Marxist model reaffirms the claim that DORSPA is broadly sufficient to ensure the existence of capitalist exploitation. The key to this extension is the plausible and empirically relevant assumption of *income-elastic* time preferences, such that the wealthy are more "patient" (and thus more willing to act as suppliers in competitive markets for productive capital) than their poorer counterparts, despite having otherwise identical preferences. The basic conclusion is strengthened by adding Marx's own assumption of exogenously given growth in the labor force. I argue moreover that the rational-choice model is better equipped than the traditional Marxist framework to provide a full understanding of the economic conditions that ensure persistent exploitation when capital accumulates.

Following a suggestion by Skillman (1995), I adapt a model developed by Ryder (1985) to this purpose. Ryder demonstrates that capital markets generate marked class polarization given heterogeneous time preferences, but does not link measures of exploitation or class-based time preferences explicitly into his account (although he notes the possibility of linking time preference to class position without changing his core conclusions). Doing so, I show, reinforces Roemer's original insight in a dramatic way: the rate of exploitation is positive in every period, and exploitation disappears in the limit only because the working class is completely immiserated. The capitalist class ends up owning all the productive wealth.

## 1. Household Consumption and Asset Holdings

In this section I summarize the key features of Ryder's model of intertemporal consumption and investment, modified to incorporate class-based differences in time preference. The reader is referred to Ryder (1985) for a full statement of the model, results, and extensions. The feature I would have the reader note is the essentially neoclassical character of the model: individuals rationally maximize utility subject to constraints given by their endowments and competitive market conditions. Where appropriate I note simplifying departures from a pure Walrasian formulation and their (lack of) substantive impact on the key results of the argument.

### *Preferences and endowments*

Imagine an economy with a single produced good in which there are  $n$  infinitely-lived households whose utility depends on their consumption streams. Time and all choice and state variables are continuously-valued. The  $i^{\text{th}}$  household's utility from a given a consumption path  $c_i(t) \geq 0$  for  $t \geq 0$  is represented by

$$(1) \quad \int_0^{\infty} e^{-\mathbf{a}_i t} U_i(c_i(t)) dt,$$

where  $U_i(c) = c^{1-\mathbf{b}} / (1-\mathbf{b})$  and  $\mathbf{a}_i > 0$ ,  $\mathbf{b} > 0$ . The parameter  $\mathbf{a}_i$  represents the  $i^{\text{th}}$  household's subjective discount rate and thus measures its degree of time preference; higher values correspond to more impatient time preferences.

Households are differentiated by their initial endowments of the produced good, denoted  $A_i$ . As such they are partitioned into subgroups  $k$  and  $l$  with respective sizes  $n_k$  and  $n_l$ , with  $A_i = A_k$  for all  $i \in k$  and  $A_i = A_l < A_k$  for all  $i \in l$ . Endowments link to

preferences in sole sense that time preferences are dependent on real income, with the household's discount rate being *inversely* related to attainable consumption.<sup>2</sup>

For convenience, this dependence is modeled in a particularly simplistic way, such that a household's discount rate depends solely on its *initial* holding of the produced good; thus  $\mathbf{a}_i = \mathbf{a}_k$  for all  $i \in k$  and  $\mathbf{a}_i = \mathbf{a}_l > \mathbf{a}_k$  for all  $i \in l$ . This restriction to reliance on initial wealth turns out not to be restrictive given household behavior. In any case, Ryder asserts that his results (summarized below) emerge with a more general specification of the dependence of time preference on real income (1985, p. 73).

Finally, all households have an endowment  $m > 0$  of labor capacity which they supply inelastically for any non-negative wage rate.

#### *Markets, constraints, and household optimization*

Households participate in two competitive markets, one for labor capacity characterized by wage rate  $w(t)$  one for capital with interest rate  $r(t)$ . All households are assumed to be price-takers and to have myopic expectations with respect to the path of wages and interest, such that current values are always assumed to hold for the future (this is a stringent, although not implausible simplifying assumption; aside from introducing the possibility of multiple equilibria, it doesn't seem that relaxing this assumption alters the basic results).

The household consumption plan implies a plan for its *asset stream*, where  $A_i(t)$  is the household's total assets at time  $t$  (a negative value if representing a debt). Given the stream of planned consumption at given interest and wage rates, assets are given by

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<sup>2</sup> Empirical support for this condition is found by Lawrance (1991).

$$A_i(t) = A_i e^{rt} + \int_0^t e^{r(t-t)} (wm - c_i(t)) dt$$

Each household must satisfy a bankruptcy constraint which dictates that the present value of planned consumption expenditures cannot exceed the present value of wage income and (possibly negative) income from the capital market. With static expectations on the wage and the interest rate, this simplifies to the condition

$$(2) \quad A_i(t) + wm/r \geq 0 \text{ for all } t \geq 0.$$

Differentiating the asset stream implies the following rule for altering the “state variable,” household wealth:

$$(3) \quad \frac{dA_i(t)}{dt} = rA_i(t) + wm - c_i(t) \text{ for all } t \geq 0.$$

The household’s problem is to choose a non-negative consumption stream to maximize (1) subject to constraints (2) and (3). This can be solved as an optimal control program including inequality constraints on the state variable. First-order conditions for an optimum yield the following expressions for the asset and consumption streams:

$$(4) \quad A_i(t) = \frac{-wm}{r} + \left(A_i + \frac{wm}{r}\right) \exp\left(\frac{r - \mathbf{a}_i}{\mathbf{b}}\right)$$

$$(5) \quad c_i(t) = \frac{(\mathbf{b} - 1)r + \mathbf{a}_i}{\mathbf{b}} \left(A_i + \frac{wm}{r}\right) \exp\left(\frac{r - \mathbf{a}_i}{\mathbf{b}}\right) = \frac{(\mathbf{b} - 1)r + \mathbf{a}_i}{\mathbf{b}} \left(A_i(t) + \frac{wm}{r}\right) \text{ for } t \geq 0.$$

For  $(\mathbf{b} - 1)r + \mathbf{a}_i \leq 0$ , a household acts as a “rational miser,” willing to postpone consumption indefinitely. However, it will turn out to be the case that

$(\mathbf{b} - 1)r + \mathbf{a}_i$  always exceeds zero after a sufficiently long horizon.

From (4), the savings of the  $t^{\text{th}}$  household is given by

$$(6) \quad s_i = dA_i(t) / dt = \left[ \min \left( 1, r - \frac{\mathbf{a}_i}{\mathbf{b}r} \right) \right] (rA_i(t) + wm).$$

Households that are patient relative to the market ( $\mathbf{a}_i < r$ ) will save, those that are impatient relative to the market ( $\mathbf{a}_i > r$ ) will dissave, and those whose discount rate just equals the market interest rate will maintain their net wealth position.

It is readily verified that increased wealth increases the net saving or dissaving of the household. The impact of changes in the market interest rate is

$$(7a) \quad \frac{\partial s_i}{\partial r} = \frac{(rA_i(t) + wm)}{\mathbf{b}r} + \frac{\mathbf{a}_i - r}{\mathbf{b}r^2} = \frac{(r^2 A_i(t) + \mathbf{a}_i wm)}{\mathbf{b}r^2}$$

for “normal” households and

$$(7b) \quad \frac{\partial s_i}{\partial r} = A_i(t)$$

for “rational misers.” A household will thus save more at higher interest rates if it either has non-negative wealth or is impatient relative to the market. As it turns out, all households eventually fall into one of the two categories.

## 2. Production and Capital Accumulation

### *Production function and aggregate inputs*

Following Ryder, I assume that total output of the manufactured commodity is given by a neoclassical production function with well-defined marginal products and constant returns to scale. Total labor input  $L$  is equal to  $nm$ . In this setting, households can hold debts or loans as well as claims on productive capital. The total capital stock is thus

equal to aggregate net asset holdings, or  $K(t) = \sum_{i=1}^n A_i(t) \geq 0$ .

To facilitate the analysis of exploitation, I depart from Ryder's setup by positing that the aggregate production is represented by a constant elasticity of substitution (CES) production function of the form

$$(8) \quad X(t) = F(L, K(t)) = \left( \mathbf{g} L^r + (1 - \mathbf{g})(K(t))^r \right)^{1/r}$$

Although restricted in form, this function allows for a wide range of cases, depending on the specification of the parameter  $\mathbf{r}$ , from perfect substitutability of inputs ( $\mathbf{r} = 1$ ) to Cobb-Douglas ( $\mathbf{r} = 0$ ) to fixed-coefficients (in the limit as  $\mathbf{r}$  approaches  $-\infty$ ).

### *Equilibrium wage and interest rates*

Perfect competition in the labor and capital markets ensures the following marginal conditions are respectively satisfied:

$$(9) \quad w(t) = F_L(L, K(t)) \text{ and}$$

$$(10) \quad r(t) = F_K(L, K(t)) \text{ for } K(t) > 0, \quad r(t) \geq F_K(L, 0) \text{ for } K(t) = 0.$$

For the CES production function, (9) and (10) imply that wages rise and the interest rate falls with capital accumulation. In addition, the ratio of interest to wage rate is given by

$$(11) \quad \frac{r(t)}{w(t)} = \frac{(1 - \mathbf{g})}{\mathbf{g}} \left( \frac{K(t)}{L} \right)^{r-1}.$$

### *Capital accumulation and its consequences*

The rate of capital accumulation is governed by saving behavior, since

$$dK(t)/dt = \sum_i dA_i(t)/dt = \sum_i s_i(r(t), A_i(t)).$$

Given the conditions on technology and time preferences, the maximum possible interest rate exceeds the discount rate of the most patient set of households, and capital accumulation occurs. If

$\mathbf{a}_k > F_K(0, K(t)) = (1 - \mathbf{g})^{(1+r)/r} (K(t))^r$ , capital accumulation drives the interest rate down to  $\mathbf{a}_k$  in the limit as  $t$  approaches infinity. For any finite time period, the interest rate exceeds the patient households' discount rate, so they earn rents in the capital market. The asymptotic level of the capital stock is  $K^*$ , satisfying the condition  $F_K(L, K^*) = \mathbf{a}_k$ . If instead  $\mathbf{a}_k < F_K(0, K(t))$ , capital accumulation is unlimited.

In either case, the relatively impatient households are completely immiserated in the limit: although the wage rate continually rises and the interest rate continually falls, their dissaving implies that income is increasingly handed over to the patient classes, until in the limit there is nothing left for consumption. The patient class accumulates all of the wealth in the economy.

### 3. Accumulation and Exploitation

#### *The rate of exploitation*

We now want to inquire what capital accumulation implies for the existence and trend in capitalist exploitation. In an economy with one manufactured commodity, labor values are redundant (which they are, at best, in any event), so that the rate of exploitation for the economy as a whole can be written

$$(12) \quad E(t) = \frac{r(t) \cdot K(t)}{w(t) \cdot L},$$

which given (11) can be expressed as

$$(13) \quad E(t) = \frac{1 - \mathbf{g}}{\mathbf{g}} \left( \frac{K(t)}{L} \right)^r.$$

The rate of exploitation is always positive, even for the limit case of Leontief technology, because otherwise the interest rate would fall to zero and even the patient

households would want to disaccumulate. Otherwise, the derivative of the rate of exploitation with respect to time has the same sign as the parameter  $r$ . Thus, for example, the rate of exploitation is constant for the Cobb-Douglas case, rising when inputs are more substitutable than in this case, and falling when less substitutable; this is a result of assuming that total labor supply is constant, and thus tendentially falling in ratio to the capital stock.

### *Labor force growth and technical change*

Our analysis has established a dramatic and durable process of exploitation in a dynamic setting with capital accumulation and unequally impatient households, such that the wealthy/patient households exploit the poor/impatient ones. But we have as yet not even included the key aspects of Marx's "general law of capital accumulation," put forward in Chapter 27, Volume I of *Capital* (1990). This analysis adds two key features to the analysis: exogenously given growth in the labor force and "capital-using, labor-saving (CU-LS)" technical change.

Ryder incorporates growth in the labor force by allowing household "size" to vary across time, and supposing that family size grows exponentially at a constant rate  $I$ . In this case, he shows that the interest rate will in every finite time period be higher than in the absence of labor force growth, and that the asymptotic interest rate is  $r^* = a_i + I$ . This is qualitatively significant in that it ensures that capital suppliers would earn rents in the capital market, even in the limit as time approaches infinity.

Finally, CU-LS technical change can be represented by reductions in the parameter  $g$ . Condition (13) shows that this leads to a higher rate of exploitation in each period.

## **Conclusion**

This paper has shown that Roemer's "rational-choice Marxist" model can be extended to incorporate capital accumulation. Contrary to the (implicit) suggestion of his critics, Roemer's central conclusion, to the effect that DORSPA is generically sufficient for the existence of exploitation, is rather dramatically borne out. The result of capital accumulation is complete immiseration of the working class. The only condition driving this result is the possibility of income-elastic time preferences, such that poorer households are more impatient. This condition is consistent with both the empirical evidence and Marx's own formulation of the problem of capital accumulation.

In hindsight, Roemer's critics do not seem to have allowed him the method of "case-appropriate abstraction" that Marx is lauded for. It is true that Roemer himself did not model the dynamic capitalist economy in Walrasian terms; but this does not mean it couldn't be done. To the contrary, if there is a methodological bias in the Walrasian framework, it is in allowing too many possibilities, rather than in restricting possible microeconomic outcomes. Addition of quintessentially Marxian postulates gives structure to the general equilibrium model and allows detailed exploration of the conditions that support persistent capitalist exploitation.

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