

**Person-Specific Labor Costs and the Employment Effects of  
Minimum Wage Policy**

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## **Abstract**

Economic debates over minimum wage policy are typically premised on the assumption that raising a binding wage floor must reduce long-run employment if the affected markets are competitive and complete. It is shown here to the contrary that employment effects of raising the minimum are indeterminate if competitive employers incur person-specific labor costs, which vary with the number of employees rather than with hours worked per employee, and there are diminishing returns in the contribution of individual hours to effective labor input. This conclusion is robust to extensions of the basic model incorporating various forms of input substitutability. In addition, the qualitative impact of varying non-labor inputs is shown to depend critically on how they interact with labor in the production function. (*JEL* D21, J23, J32, J38)

The employment consequences of minimum wage legislation have been the subject of renewed attention in the labor economics literature, due both to recent policy initiatives and to controversial new evidence suggesting instances of zero or even positive employment effects from increasing wage floors. David Card and Alan Krueger, who have contributed significantly to this literature, discuss the relevant issues and evidence in their recent and controversial work, *Myth and Measurement* (1995).

Both Card and Krueger and their critics (see, for example, the *Industrial and Labor Relations Review* review symposium devoted to their book (Ehrenberg, 1995)) argue as though the sides of the debate are dictated by a strict analytical dichotomy with respect to the structure of labor markets, such that increasing an effective wage floor necessarily reduces long-run employment if the affected markets are competitive and complete. As Card and Krueger note, it is readily demonstrated that imposing or raising a binding wage floor may increase employment if firms enjoy monopsony power, or if labor markets are characterized by asymmetric information with respect to work effort (Rebitzer and Taylor, 1995) or the quality of job applicants (Stiglitz, 1987). But what if none of these departures from ideal market conditions obtain? Does it *necessarily* follow that competitive firms reduce long-run employment when a wage floor is raised?

This note answers that question in the negative given that the affected markets are characterized by *quasi-fixed* or *person-specific* labor costs,<sup>1</sup> which vary with the number of employees rather than the number of hours worked. Person-specific labor costs arise, for

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<sup>1</sup> Although the former term is used more frequently in the literature (dating back at least to Oi (1962)), the latter is given preference here as being more transparently descriptive of the phenomenon under study.

example, in providing employees with offices or other work spaces, certain fringe benefits, sanitary facilities and other workplace amenities, specific training, and equipment. These costs are evidently pervasive, and they often constitute a significant proportion of total labor costs (Hart, 1984).

Faced with person-specific costs, profit-seeking employers are not indifferent to the mix of *number of workers* and *hours per worker* utilized in generating given magnitudes of effective labor input. Donaldson and Eaton (1984) study the implications of such costs for firms' choice of hours, labor force size, and employment contract provisions given diminishing marginal returns of individual labor hours to effective labor inputs. However, they do not consider the implications of minimum wage policy in the presence of person-specific labor costs.

Based on a simplified version of their model, it is demonstrated here that increasing a binding minimum wage will lead employers to substitute employees hired for hours worked by each employee, resulting in an increase in employment for any given quantity of desired labor input. In addition, the utility of any worker who remains employed must increase, although wage income per employee may decline.

The *net* change in employment is determined by combining the positive substitution effect and the more familiar negative scale effect of increasing unit labor costs, the sum of which cannot be signed *a priori*. Consequently, under the stated conditions, the conventional view that raising the minimum wage lowers employment (measured by the number of workers hired) cannot be considered a presumptive, let alone a necessary, outcome of competitive market behavior.

## I. A Basic Model of Labor Demand with Person-Specific Costs

Imagine that a profit-maximizing firm supplies a competitive product market using a technology summarized by the twice continuously differentiable production function  $x = G(l)$ , where  $x$  denotes quantity of output,  $l$  represents units of “effective” labor input, and  $G$  is strictly increasing and strictly concave in its argument. Let  $p$  denote output price.

Effective labor input depends on the number of workers  $n$  and hours per worker  $h$  according to the multiplicatively separable function  $l = A(h) \cdot n$ . Assume that  $A$  is thrice continuously differentiable, strictly increasing, and strictly concave in  $h$ , suggesting diminishing marginal returns to increasing an individual’s work hours.<sup>2</sup>

Note that although the production function is strictly concave in  $l$ , it may not be concave in the constituent elements  $n$  and  $h$ . Consequently, standard concave programming analysis may not be validly applied to the firm’s profit-maximization problem with respect to these choice variables. One method of dealing with this difficulty, utilized by Donaldson and Eaton and emulated below, is to break the problem into two parts, involving first cost minimization with respect to hours and then profit maximization given the resulting minimized unit cost of labor.

The firm faces two types of costs in engaging labor inputs. First and familiarly, its labor costs vary with hours worked; in the standard model of competitive labor markets the marginal cost of hours worked is simply given by the market wage rate  $w$ . In addition, however, the firm incurs a *person-specific* cost  $c$  per worker hired, regardless of the hours expended per worker.

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<sup>2</sup> This is slightly stronger than necessary; it is only required that the function  $A(h)$  exhibit *eventually* diminishing returns.

For the sake of simplicity in deriving the main result, non-labor inputs in production, work effort, and unit person-specific labor costs  $c$  are initially treated as constant. The consequences of taking each of these as choice variables are considered in section V.

Since the firm is not indifferent as to how effective labor inputs are provided when faced with person-specific labor costs, it may be inappropriate to depict the competitive labor-buying firm as a “wage-taker” who allows its workers freely to determine their respective working hours. For each worker, the firm instead chooses a bundle of income and hours worked from a frontier derived from the worker’s preferences and reservation utility level. Assuming for a moment that the minimum wage constraint is not binding on the firm’s choices, and letting  $y$  represent compensation per employee, its labor supply constraint is

$$(1) \quad U(y, \bar{t} - h) \geq \bar{u} ,$$

where  $U$  is a twice continuously differentiable, strictly quasiconcave utility function in income and leisure,  $\bar{t}$  is the endowment of hours and  $\bar{u}$  the reservation level of utility for each prospective employee. Assume that  $U$  is strictly increasing in income and leisure.

The labor supply constraint can be more conveniently written as

$$(2) \quad y \geq Y(h) ,$$

where the worker’s income-hours frontier  $Y(h)$  (equivalent to the indifference curve in  $(h, y)$  space corresponding to utility level  $\bar{u}$ ), given the foregoing assumptions, is twice continuously differentiable, strictly increasing and strictly convex in  $h$ . Without loss of generality, suppose the reservation level of utility ensures that  $Y(h) > 0$  for all  $h > 0$ .

Note, again ignoring the minimum wage constraint, that the firm's desire to maximize profits ensures that the labor supply constraint (2) will hold with equality. The firm's total labor costs are therefore given by the expression  $(c + Y(h)) \cdot n$ , and the firm's optimizing choice of labor hours corresponds to a point such as that illustrated at point A in Figure 1, involving a tangency between the income-hours frontier and the firm's highest attainable isoprofit curve  $\mathbf{p}^*$ .

Donaldson and Eaton study this problem in detail.

**[Figure 1 about here]**

One implication of their analysis is that the firm enjoys some leeway in its choice of a compensation scheme to implement the optimal outcome. The firm might for example pay workers by a straight hourly wage  $w^* = Y(h^*)/h^*$ , corresponding to line OB in Figure 1, but with the consequence that each worker's desired marginal tradeoff between hours and income differs from that of the firm, so that the latter might be compelled to specify hours inputs by contract. Alternatively, the firm might offer a two-part compensation scheme such as OCD in Figure 1, which aligns firm and worker incentives at the margin.

But now suppose there exists a minimum wage policy which requires that effective hourly compensation not fall below some legislatively determined floor. This introduces an additional constraint of the form

$$(3) \quad \frac{y}{h} \geq w_f,$$

where  $w_f$  is the stipulated wage floor. If the wage floor is effective, then  $w_f > w^*$ , and the new constraint, illustrated by line OF in Figure 1, is binding, with a constrained optimum for the firm such as at point E. The original labor supply constraint will typically not be relevant, but even if

it were, the firm's total labor costs now take the form  $C(h, n; c, w_f) = (c + w_f h) \cdot n$ .<sup>3</sup> Note that each employed worker receives economic rent under the minimum wage except for the boundary case in which both constraints bind simultaneously. The firm's optimization problem given the minimum wage constraint is explored in the next two sections.

## II. The firm's labor cost minimization problem

Consider first the firm's problem of minimizing the cost of a given quantity of effective labor input  $\tilde{l}$ . The firm seeks to minimize  $C(h, n; c, w_f)$  subject to

$$(4) \quad A(h) \cdot n = \tilde{l}.$$

This is equivalent to choosing  $h$  to minimize unit labor cost, equal to

$$(5) \quad B(h; c, w_f) = \frac{c + w_f \cdot h}{A(h)},$$

and then choosing  $n$  to satisfy (4).

Assume that a strictly interior solution to this programming problem obtains. The constrained optimum  $\bar{h}$  must satisfy the corresponding first-order condition

$$(6) \quad B'(\bar{h}) = \frac{A(\bar{h}) \cdot w_f - (c + w_f \cdot \bar{h})}{[A(\bar{h})]^2} = 0.$$

The function  $B(h)$  is strictly pseudo-convex, which is to say that given  $B'(\bar{h}) = 0$ ,  $B$  is strictly convex in a neighborhood of  $\bar{h}$ , so that

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<sup>3</sup> A semicolon is used in defining objective functions to separate choice variables from parameters. The latter may be suppressed in subsequent discussion if no ambiguity is thereby created.

$$(7) \quad B''(\bar{h}) = \frac{-(c + w_f \bar{h}) A''}{[A(\bar{h})]^2} > 0$$

by virtue of the strict concavity of  $A$ . As a consequence, fulfillment of the first-order condition (6) is sufficient for  $\bar{h}$  to be a local optimum, assuming an interior solution.

On the basis of (6) and (7) the standard apparatus of comparative static analysis can be applied. Invoking the implicit function theorem, we can write  $\bar{h} = H(c, w_f)$  such that

$$(8) \quad B'(H(c, w_f); c, w_f) \equiv 0$$

for some neighborhood of initial values of the parameters. Totally differentiating this identity with respect to  $w_f$  and rearranging yields

$$(9) \quad \frac{\partial \bar{h}}{\partial w} = \frac{-(A(\bar{h}) - \bar{h} \cdot A'(\bar{h}))}{[A(\bar{h})]^2 \cdot B''(\bar{h})},$$

which is strictly negative by the first- and second-order conditions (6) and (7).<sup>4</sup>

From (4) and (9) it follows that

$$(10) \quad \frac{\partial \bar{n}}{\partial w} > 0,$$

where  $\bar{n}$  is the cost-minimizing number of workers hired for a given level of desired labor input

$\tilde{l}$ . Finally, a procedure similar to that followed above establishes that  $\partial \bar{h} / \partial c > 0$ , as might be expected intuitively.

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<sup>4</sup> Given an effective wage floor and person-specific labor costs, the model yields an unambiguous test for the assumption of (eventually) diminishing marginal returns in the contribution of individual hours to effective labor input (i.e., that  $A'' < 0$ , at least at the optimum). It is readily shown that in the absence of diminishing returns, the reservation utility constraint (1) on individual labor supply is always binding if the wage floor is, with the implication that the firm *increases* rather than decreases hours per worker in response to the imposition or elevation of a binding minimum wage rate.

The economic logic of results (9) and (10) is illustrated in Figure 2. The effect of an increase in the minimum wage is to increase (in absolute value) the slope of labor isocost curves at any given  $(h,n)$  combination, which induces the firm to substitute employees for hours worked per employee. Consequently, the number of employees engaged to generate any given level of effective labor input unambiguously increases.

**[Figure 2 about here]**

Expressing (9) as an elasticity gives

$$(11) \quad \mathbf{h}_{hw} = \frac{c / (c + w_f \cdot \bar{h})}{\mathbf{a}(\bar{h})} < 0,$$

where  $\mathbf{h}_{hw} = \frac{w_f}{\bar{h}} \cdot \frac{\partial \bar{h}}{\partial w}$  is the wage elasticity of the firm's cost-minimizing choice of labor hours

per worker, evaluated at the wage floor, and  $\mathbf{a}(h) = \frac{A''(h) \cdot h}{A'(h)}$  is an elasticity expression

indicating the degree of diminishing returns in the per-capita effective labor function  $A(h)$ .

Similar calculations show that the elasticity of cost-minimizing hours with respect to changes in  $c$ , denoted  $\mathbf{h}_{hc}$ , is just equal to  $-\mathbf{h}_{hw}$ .

Allowing  $\bar{h}$  to vary optimally, the absolute value of the elasticity expressed in (11) is non-decreasing in  $c$  and non-increasing in  $w_f$  if  $\mathbf{a}$  is non-decreasing in  $h$  and  $|\mathbf{h}_{hw}| \leq 1$  and *only if* one or the other of the latter two conditions holds. For example, if  $A(h) = h^b$ ,  $b \in (0,1)$ , then  $\mathbf{a}(h) = (b - 1)$  and  $\mathbf{h}_{hw} = -1$  for all values of  $c$  and  $w_f$  such that the firm desires to produce.

Since the cost-minimizing quantity of labor hours  $\bar{h}$  is independent of  $p$  and  $\tilde{l}$ , the minimized level of unit labor cost can be written  $q = Q(c, w_f)$ . This is the effective unit cost which determines the firm's profit-maximizing choice of labor, considered next.

### III. Implications of the Firm's Profit-Maximizing Labor Choice

Given the cost-minimizing level of unit labor cost  $q$ , the firm chooses the level of total labor input  $l$  to maximize profit, expressed as

$$(12) \quad \Pi(l; q, z) = pG(l) - ql - z,$$

where  $z$  denotes fixed non-labor costs. Since  $G$  is strictly concave, the optimal choice  $l^*$  is unique. Again assuming an interior solution, the necessary and sufficient condition for an optimum is

$$(13) \quad \Pi_l(l^*; q) = pG'(l^*) - q = 0.$$

By the implicit function theorem, the optimum can be written as a differentiable function  $l^* = L(p, q)$  for some neighborhood of the initial values of  $p$  and  $q$ . Substituting this function into (13) transforms the expression into an identity. Total differentiating this identity with respect to  $q$  gives

$$(14) \quad \frac{\eta l^*}{\eta q} = \frac{1}{pG''},$$

which is negative as might be expected.

Using the first-order condition (13), the corresponding elasticity expression for (14) can be written

$$(15) \quad \mathbf{h}_{lq} = \frac{q}{pG'' \cdot l^*} = (\mathbf{g}(l^*))^{-1} < 0,$$

where  $\mathbf{h}_{lq} = \frac{\% \ln l^*}{\% \ln q}$ , and similar to  $\mathbf{a}(h)$ ,  $\mathbf{g}(l^*) = \frac{G'' l^*}{G'}$  is an elasticity measure of the degree

of diminishing returns, in this case with respect to the production function  $G$ . It is readily

checked that  $d\mathbf{h}_{lq}/dq$  has the same sign as  $\mathbf{g}'(l)$ , so that, for instance,  $\mathbf{h}_{lq}$  becomes less

negative as  $q$  increases if  $\mathbf{g}'(l) > 0$ . Moreover, comparative static analysis parallel to the

foregoing reveals that  $\mathbf{h}_{lp} = \frac{\% \ln l^*}{\% \ln p}$ , an elasticity expression representing the responsiveness of

individual labor demand to changes in output price, is just equal to  $-\mathbf{h}_{lq}$ .

It's now possible to consider the total impact of an increase in the minimum wage on the profit-maximizing size of the firm's labor force. Recall that minimized unit cost  $q$  is itself a function of  $w_f$ , as is the cost-minimizing magnitude of hours per worker  $\bar{h}$ . Based on equation (4) and the foregoing optimization conditions, the firm's quantity demanded of workers (ignoring that its labor force must be measured in integers) can be written

$$(16) \quad n^* = N(c, w_f) = \frac{L(Q(c, w_f))}{A(H(c, w_f))}$$

Totally differentiation of (14) with respect to  $w_f$ , evaluated at the profit-maximizing solution while holding output price  $p$  constant, gives

$$\frac{\% n^*}{\% w} = \frac{A \cdot \frac{\% l^*}{\% q} \cdot \frac{\% q}{\% w} - l^* A' \cdot \frac{\% h^*}{\% w}}{A^2} = \frac{\frac{\% l^*}{\% q} \cdot \frac{h^*}{A} - l^* \cdot \frac{w_f}{c + w_f h^*} \cdot \frac{\% h^*}{\% w}}{A}$$

(applying the envelope theorem with respect to  $q$  and first-order condition (6)), yielding, after some substitution and rearrangement,

$$(17) \quad \mathbf{h}_{nw} = \frac{\mathcal{J} n^*}{\mathcal{J} w} \cdot \frac{w_f}{n^*} = \mathbf{w}(\mathbf{h}_{lq} - \mathbf{h}_{hw}),$$

where  $\mathbf{w} = \frac{w_f h^*}{c + w_f h^*}$ , the ratio of wages to total labor cost. Expression (17) is a relative of

the well-known Slutsky equation with respect to household demand. It shows that the net effect of an increase in the wage floor on the number of workers demanded by the firm, expressed as an elasticity, is the sum of a negative *labor-specific scale effect*  $\mathbf{w}\mathbf{h}_{lq}$  and a positive *labor-specific substitution effect*  $-\mathbf{w}\mathbf{h}_{hw}$ .

Note critically that in the presence of person-specific labor costs, the net effect on firm-level employment of an increase in the minimum wage cannot be signed *a priori* without further conditions. This is also true with regard to the impact of  $c$  on the magnitude of the employment elasticity. For example, if negative,  $\mathbf{h}_{nw}$  is increasing in  $c$  if in addition  $\mathbf{a}'(h)$  and  $\mathbf{g}'(l)$  are non-negative and  $|\mathbf{h}_{hw}| \leq 1$ .

Note further that so long as  $\mathbf{h}_{nw} \geq 0$ , the effect of raising the wage floor is unambiguously (at least in the context of partial equilibrium analysis) to redistribute welfare from capital to labor, so that workers employed under the original wage floor receive higher economic rents. However, labor income per worker, given by  $y^* = w_f h^*$ , may decline with an increase in the minimum wage. There is in fact a direct tradeoff between the firm-level employment and income effects of a wage floor hike: other things equal, the larger in absolute value is the wage elasticity of hours demand  $\mathbf{h}_{hw}$ , the larger is the employment effect and the smaller is the change in wage

income per worker. Of course, the latter change is negative for absolute values of the elasticity greater than one.

#### IV. Industry-Level Employment Effects of Raising the Minimum Wage

The employment effect recorded in expression (17) is derived treating output price  $p$  as a parameter. However, if most or all firms in a given product market find their costs increased when a wage floor is raised, one would expect a corresponding increase in output price and a consequent reduction in equilibrium market output. Therefore net employment effects of increasing the minimum wage must be calculated at the industry level. Note, however, that the term “industry” should be interpreted loosely, since it is possible that workers covered by minimum wage legislation may be employed in the production of a wide array of goods.

If output price is understood to vary with the wage rate, the *firm-level* employment effect of raising the minimum wage becomes

$$(18) \quad \mathbf{h}_{mw}^p = \frac{d n^*}{d w} \cdot \frac{w_f}{n^*} = \mathbf{w} \left( (1 - \mathbf{h}_{pq}) \mathbf{h}_{lq} - \mathbf{h}_{hw} \right)$$

(making use of the result that  $\mathbf{h}_{lp} = -\mathbf{h}_{lq}$ , and using the total rather than the partial derivative

operator with respect to  $n^*$  to indicate the endogeneity of  $p$ ), where  $\mathbf{h}_{pq} = \frac{d \ln p_l}{d \ln q} \geq 0$

measures the responsiveness of industry equilibrium price  $p_l$  to changes in labor cost  $q$ . This elasticity measure embodies two separate considerations, the impact of rising  $q$  on the firm’s average cost, and the response of equilibrium price to this change in cost. For example, a non-discriminating monopolist will, other things equal, increase its price less than a competitive industry in response to a given cost increase.

Note that equation (18) is larger in value than its counterpart in (17), if all common terms are equated. This is the case because each firm which remains in the affected industry will, *ceteris paribus*, increase its output, and consequently its demand for labor, in response to the price increase occasioned by the higher minimum wage. However, increasing the wage rate will also typically alter the number of firms in a given market, so these industry-level changes must also be taken into account.

To accomplish this, assume for convenience that all affected firms are identical and operate in the same industry, and thus write industry-level employment as  $n_I = m_I \cdot n^*$ , where  $n^*$  as before denotes each firm's profit-maximizing demand for employees, and  $m_I$  represents the equilibrium number of firms in the industry. This can in turn be expressed as the ratio of equilibrium quantity demanded in the market to equilibrium firm output, or

$$(19) \quad m_I = \frac{X(P(q))}{G(L(P(q), q))},$$

where  $x_I = X(p)$  denotes market quantity demanded as a function of the equilibrium price, and  $p_I = P(q)$  denotes equilibrium market price as a function of labor cost.

Given (19), the industry-level employment effect of raising the minimum wage is

$$\frac{dn_I}{dw} = m_I \cdot \frac{dn^*}{dw} + n^* \cdot \frac{dm_I}{dw} \text{ (ignoring that numbers of firms and employees can in practice take}$$

only integer values), which after some substitution and rearrangement yields

$$(20) \quad \mathbf{h}_{nw}^I = \frac{dn_I}{dw} \cdot \frac{w_f}{n_I} = \mathbf{w} \left[ \mathbf{h}_{pq} \cdot \mathbf{h}_{xp}^I + (1 - \mathbf{h}_{pq}) \frac{(p_I \cdot x^* - ql^*) \mathbf{h}_{lq}}{p_I \cdot x^*} - \mathbf{h}_{hw} \right]$$

(again using total rather than partial derivative operators to indicate that both direct and indirect effects on employment arise when output price is endogenous), where  $h_{xp}^I$  is the elasticity of market demand, assumed for the sake of discussion to be strictly negative, and other variables are defined as before. The labor-specific scale effect defined earlier is shown to correspond at the industry level to a weighted average of two negative terms governed by essentially different microeconomic considerations informing the elasticities of firm-level labor demand and market-level product demand.

As with its firm-level counterparts in (17) and (18), the net industry-level employment effect expressed in (20) cannot be signed *a priori*. Thus, depending on specific technological and market conditions, increasing the minimum wage may reduce, increase, or leave unchanged total employment of workers affected by the change.

## V. Extensions of the Basic Model

The paper's central result, expressed in equation (20), that the net employment effect of raising the minimum wage cannot be determined *a priori* when person-specific labor costs are present, was derived on the basis of a simple model in which non-labor inputs, work effort, and the magnitude per worker of person-specific costs are in effect held constant. This qualitative assessment is not affected by incorporating these elements as choice variables in the firm's optimization problem. However, as indicated below, doing so potentially alters the magnitude and even the sign of the employment effect.

*Variable non-labor inputs*

Suppose we represent non-labor inputs in production by introducing a new argument  $k$ , “capital services,” into the production function, thus writing  $x = G(l, k)$ . Let  $r$  denote the parametric unit price of capital services. Assume that  $G$  is increasing and strongly concave in its arguments (so that the corresponding Jacobian matrix is negative definite for all positive values of the inputs), and indicate first and second partial derivatives by subscripts, with  $G_i$  denoting the marginal product of input  $i$  and  $G_{ij}$  the marginal impact of increasing input  $j$  on the marginal product of  $i$ ,  $i, j = l, k$ .

Let the notation used in the basic model be correspondingly modified so that  $\mathbf{g}_{ij}(l, k)$  represents an elasticity expression measuring the responsiveness of  $i$ 's marginal product to changes in input  $j$ . When  $i=j$ , this expression is interpreted similarly to  $\mathbf{g}$  as a measure of the degree of diminishing returns. When  $i \neq j$ , this is instead a measure of the degree of complementarity among factors. (Note that  $\mathbf{g}_{jk}$  and  $\mathbf{g}_{kl}$  are not necessarily equal, but must have the same sign.)

In this scenario the firm chooses inputs  $l$  and  $k$  to maximize

$$(21) \quad \Pi(l, k; q, r) = pG(l, k) - ql - rk .$$

Standard optimization analysis yields, with some substitution and rearrangement,

$$(22) \quad \mathbf{h}_{lq}^k = \left( \mathbf{g}_{ll} - \frac{\mathbf{g}_{lk}\mathbf{g}_{kl}}{\mathbf{g}_{kk}} \right)^{-1} ,$$

where the superscript  $k$  indicates that the firm's elasticity of effective labor demand is now derived taking the capital input as variable. The term in parentheses is guaranteed to be negative by the strong concavity of  $G$ . Comparison with the corresponding expression (15)

shows that the newly derived elasticity is larger in absolute value (i.e., more negative) than its “short-run” version, if  $g_{ll}$  is equated to  $g$ , its counterpart in the basic model.

The counterpart to expression (20) becomes much more complicated when there are multiple variable inputs in production. However, for the special case of constant returns to scale technology with two inputs (i.e., given that  $G$  is homogeneous of degree one in inputs  $l$  and  $k$ ), a familiar characterization of long-run labor demand elasticity (see, for example, Hamermesh (1993), p. 24) can be adapted to yield

$$(23) \quad h'_{nw} = w \left[ (l h'_{xp} + (1-l)s) - h'_{hw} \right],$$

where  $s$  is the elasticity of substitution of  $k$  for  $l$  and  $l$  is labor’s share of total production cost.

Note that the sign of the net employment effect remains indeterminate.

Moreover, a subtle distinction must be noted with respect to the impact of variability in non-labor inputs. As suggested by the examples given in the introduction, certain of these inputs may vary directly with the number of workers employed and thus contribute to unit person-specific costs  $c$ . This is particularly significant if these influence the marginal contributions to effective labor of individual hours, the case considered next.

#### *Variable unit person-specific costs*

Now consider a scenario in which the magnitude of unit person-specific cost  $c$  is itself a choice variable which influences the effective labor contribution per worker.<sup>5</sup> Accordingly, let

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<sup>5</sup> Firm-specific training activities are an obvious example of this phenomenon. Other possibilities: specialized clothing might render service personnel more immediately identifiable and thus less likely to be impeded in performing their work; proximate sanitary facilities may reduce undue interruptions; on-site lockers may reduce startup costs. Outside the framework of competitive and complete markets, search and screening expenditures may increase the average quality of workers, while person-specific monitoring may boost individual work effort.

this contribution be written  $A(h, c)$  and assume that it is strongly concave in its arguments. Let subscripts be used to represent partial derivatives, so that  $A_i$  denotes the marginal contribution of variable  $i$  to a worker's effective labor input and  $A_{ij}$  represents the effect of increasing labor variable  $j$  on the marginal contribution to effective labor of variable  $i$ ,  $i, j = h, c$ , supposing that  $A_{hc} \geq 0$ . Since  $c$  thus provides benefits as well as costs to the firm, there is no loss in assuming that  $A$  is strictly increasing in its arguments.

Let the notation used in the basic model be modified accordingly so that  $\mathbf{a}_{ij}$  denotes an elasticity measure of the responsiveness of argument  $i$ 's marginal contribution to individual labor input to changes in argument  $j$ . Thus, as with the term  $\mathbf{a}$  in the basic model,  $\mathbf{a}_{ii} < 0$  measures the intensity of diminishing returns to argument  $i$ , while  $\mathbf{a}_{ij} \geq 0$ ,  $j \neq i$ , is a measure of the degree of complementarity between the two arguments.

The firm in this case chooses both  $h$  and  $c$  in minimizing  $B(h, c; w_f) = \frac{c + w_f h}{A(h, c)}$ . Standard

analytical procedures, again assuming an interior solution, yield

$$(24) \quad \mathbf{h}_{hw}^c = \frac{(1 - w) \left( 1 + \frac{\mathbf{a}_{ch} \cdot w_f}{\mathbf{a}_{cc}} \right)}{\mathbf{a}_{hh} - \frac{\mathbf{a}_{ch} \mathbf{a}_{hc}}{\mathbf{a}_{cc}}},$$

where the superscript- $c$  indicates that unit person-specific costs are taken as variable.

Noting the correspondence of  $\mathbf{a}_{hh}$  and  $\mathbf{a}$  and that  $A_{cc} < 0$ , the sign of (24) cannot be determined *a priori*, nor can its magnitude be unambiguously compared with that of (11), its counterpart in the simpler model. However, if  $A_{ch} = 0$ , implying in turn that  $\mathbf{a}_{ij} = 0$ ,  $i \neq j$ , then

this term is strictly more elastic than its counterpart in the basic model, if corresponding terms are equated.

It can by parallel steps be shown that

$$(25) \quad \frac{\bar{c} \cdot \mathbf{h}_{cw}}{w_f \cdot \bar{h}} = \frac{d\bar{c}/dw}{\bar{h}} = \frac{(\mathbf{w} - 1) \left( \frac{\mathbf{a}_{hh}}{\mathbf{a}_{cc}} + \frac{\bar{c} \cdot \mathbf{a}_{ch}}{\bar{h} \cdot \mathbf{a}_{cc}} \right)}{\mathbf{a}_{hh} - \mathbf{a}_{ch} \mathbf{a}_{hc} / \mathbf{a}_{cc}},$$

which is similarly indeterminate in sign. Consequently the impact of raising the wage floor on productivity-enhancing person-specific labor costs is also ambiguous. Note, however, that  $\bar{c}$  is strictly increasing in  $w_f$  if  $A_{ch} = 0$ .

Evaluated at the profit-maximizing solution, the net impact of raising the minimum wage on firm-level employment in this scenario, again treating output price  $p$  as parametric, can be expressed as

$$(26) \quad \mathbf{h}_{nw}^c = \frac{\mathcal{I} n^*}{\mathcal{I} w} \cdot \frac{w_f}{n^*} = \mathbf{w} \left( \mathbf{h}_{lq} - \left( \mathbf{h}_{hw}^c + \frac{c^* \cdot \mathbf{h}_{cw}}{h^* w_f} \right) \right).$$

In general, neither the sign of expression (26) nor its magnitude relative to that of (17) can be established. However, one can verify using (24) and (25) that, other things equal, expression (26) is larger in magnitude (i.e., less negative) than (17) if  $A_{ch} = 0$ . In any event, the net employment effect of raising the minimum wage remains indeterminate.

### *Work effort*

Finally, suppose the model were altered to incorporate a third dimension of labor input, work effort  $e$ , and thus write effective labor contribution per worker as  $A(h, e)$ . Let  $A$  be

strongly concave in its arguments, with  $A_{he} \geq 0$ . Assume further that  $A$  is strictly increasing in  $h$ , and increasing in work effort for values of  $e \in (0, \tilde{e}]$ .

Suppose that the firm optimizes with respect to  $h$ ,  $e$ , and  $l$ . The counterpart to (16) representing the dependence of profit-maximizing employment on the parameters is

$$(27) \quad n^{e*} = N^e(c, w_f) = \frac{L(Q(c, w_f))}{A(H(c, w_f), E(c, w_f))},$$

where the superscript  $e$  indicates that  $n^*$  is determined given that  $e$  is a choice variable, and  $e^* = E(c, w_f)$  indicates profit-maximizing effort as an implicit function of the parameters. This implies the comparative static expression

$$(28) \quad \frac{dn^{e*}}{dw} = \frac{A \cdot \frac{dl^*}{dq} \cdot \frac{\partial q}{\partial w} - l^* \left( A_h \cdot \frac{\partial h^*}{\partial w} + A_e \cdot \frac{\partial e^*}{\partial w} \right)}{A^2}.$$

Note that there is no direct or indirect price of effort to the firm, so long as the worker's reservation utility is more than satisfied by the terms of employment. As a result, the firm may, *if it wishes*, and contractual conditions permit, set effort so that the reservation utility constraint is just binding. Consequently, two cases must be considered in evaluating expression (28).

In the first case, suppose that the marginal contribution of effort to effective labor input reaches zero (that is,  $A_e = 0$ ) before the reservation utility constraint is engaged. In this case, adjusting work effort so that each worker just receives the reservation utility level is not optimal, so cost minimization implies that  $A_e = 0$  at the optimum, since the inframarginal cost of effort to the firm is zero. In addition, standard comparative static analysis can be invoked to demonstrate that

$$(29) \quad \mathbf{h}_{hw}^e = \frac{(1-w)}{\mathbf{a}_{hh} - \frac{\mathbf{a}_{he}\mathbf{a}_{eh}}{\mathbf{a}_{ee}}}$$

is strictly negative due to the strong concavity of  $A$  in its arguments. Comparison of (29) to its counterpart in (11) demonstrates that the negative responsiveness of hours to an increase in wages is unambiguously *greater* when effort is also a choice variable, other things equal, given  $A_e = 0$  and  $A_{eh} > 0$ . This implies, via (28), with other things equal, that allowing optimal variation in effort increases (i.e. makes more positive or less negative) the value of  $\mathbf{h}_{nw}$ , thus expanding the scope for employment gains.

Suppose alternatively that  $\tilde{e}$  exceeds the magnitude of work effort at which each worker receives just the reservation utility for a given combination of wage rate  $w_f$  and hours  $h$ . This implies that the inframarginal contribution of work effort to effective labor input is strictly positive, and the reservation utility constraint will consequently be binding at the optimum.

Accordingly, let  $\bar{e} = \bar{E}(h, w_f)$  denote the level of work effort which affords the reservation level of utility given the minimum wage and hours expended per employee. It is readily verified that  $\bar{e}$  is increasing in  $w_f$  and decreasing in  $h$  given that the minimum wage constraint is also binding.

In this case, allowing effort to vary has an ambiguous impact on the firm-level employment effect of raising the minimum wage. Evaluated at the profit-maximizing solution, expression (28) takes the specific form

$$(30) \quad \mathbf{h}_{nw}^{\bar{e}} = \frac{dn^*}{dw} \cdot \frac{w_f}{n^*} = \mathbf{w} \left( \mathbf{h}_{lq} - \left( \mathbf{h}_{hw}^{\bar{e}} + \frac{A_e q}{h^*} \cdot \frac{d\bar{e}}{dw} \right) \right),$$

where  $h_{hw}^{\bar{e}}$  differs indeterminately from  $h_{hw}^e$  as expressed in (28). Consequently the precise impact of allowing work effort to vary cannot be determined in this scenario.

In sum, incorporating variable work effort in the model does not alter the indeterminacy of employment effects, and make increase the scope for positive employment effects from raising the wage floor, other things equal.

## **VI. Discussion and Conclusion**

Opponents of minimum wage policies have often, if not typically, argued as though it were presumptive that raising a wage floor must curtail employment, particularly in the long run. Of course, no such presumption can be established on theoretical grounds if the affected labor markets are subject to certain imperfections. It is shown here that in addition negative employment effects may not obtain even if the affected labor markets are competitive and complete, given that they are characterized by person-specific labor costs and diminishing returns to effective labor inputs of individual work hours. Under the stated conditions, the conventional wisdom may in fact be completely reversed, with employment rising and income per worker falling as a result of a hike in the wage floor.

These results are shown to be robust to extensions of the basic model which incorporate various forms of input substitutability. Moreover, under the conditions studied here raising a wage floor may encourage more rather than less productivity-enhancing expenditures per worker, and may additionally serve as a policy instrument in allocating work hours more evenly across an economy's labor force.

At least in a partial equilibrium context, it remains the case that raising a binding wage floor must incur additional dead-weight losses, although there will be an unambiguous redistribution of welfare from capital to labor so long as the employment effect is non-negative. Workers who remain employed under a minimum wage regime receive economic rents unless firms are willing and able to adjust individual effort levels to eliminate them. In a general equilibrium context, however, unambiguous efficiency judgments cannot be made given that there is any other deviation from ideal conditions.

In light of the results presented here, the impact of minimum wage policy on employment is essentially an empirical issue, with no presumption assignable to any particular market response. Furthermore, employment effects may vary across economies or over time as technologies or market conditions change. Of course, the results presented here cannot of themselves establish a case for wage floors as an instrument of government policy with respect to poverty or income inequality, and it may be that the contingent nature of employment effects argues against a universal minimum wage policy. However, the considerations studied here may serve to reset the terms of ongoing debates and provide a basis for more nuanced investigations of this policy instrument.

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