

Capital Accumulation, Technical Change, and Profit Rate Dynamics

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1. Introduction

The purpose of this paper is to explore issues raised by Marx's analysis of capitalist dynamics in Volume I, Part 7 and Volume III, Part 3 of *Capital*. This investigation is conducted using a variable-proportions, constant-returns (i.e., Cobb-Douglas) production technology. My justification for this procedure, seemingly at odds with Marx's characterization of capitalist production in Volume I, chapters 13-15, is two-fold: first, these production conditions can be shown to provide a plausible and tractable basis for assessing fundamental Marxian claims concerning the logic of capitalist accumulation, and second, nothing in Marx's account appears to rule out the possibility of variable-proportions technology (and to do so seems excessively restrictive). Possible implications of the centralization of capital, emphasized by Marx, are discussed in the conclusion.

The paper primarily addresses the tension between Marx's assessment of the consequences of capital accumulation in Volume I, chapter 25, and that developed in Volume III, Chapters 13 and 14. Specifically, the former derives conditions which might be expected at minimum to maintain a stable if not increasing center of gravity for the average rate of profit in a capitalist economy, while the latter suggests a "tendency of the rate of profit to fall." The analysis presented here resolves the conflict in favor of Marx's Volume I account, demonstrating that

technical changes of the sort Marx emphasized can be understood to depress the rate of profit only in a very indirect sense.

The paper also confirms an important aspect of Marx's criticism of vulgar political economy by demonstrating the possibility of persistently positive rates of profit in the absence of any opportunity costs of productive "risk-taking" or "abstinence." In particular, the analysis shows that *relative scarcity* of capital, necessary (and in John Roemer's (1982) reformulation of Marxian economics, generically sufficient) for the existence of positive profits, is guaranteed by the logic of capitalist accumulation even in the absence of such opportunity costs, supporting Marx's implicit claim that capitalist profit represents pure economic rent.

2. Model

Consider a capitalist economy with an indeterminate time horizon, where time proceeds by discrete periods indexed by $t = 0, 1, 2, \dots$. Let there be F identical firms and L_t identical workers at time t . There is one produced good, which is also used as an input to production. The duple (k_t, l_t) denotes the (homogeneous) inputs of the produced good and labor power (assuming a particular solution to the labor extraction problem) respectively used by a representative firm in production. The production technology is given by a Cobb-Douglas production function of the form $x = k_t^a l_t^{1-a}$, $a \in (0,1)$. Production is understood to take time: in particular, the use of the produced good as an input in the current period requires its production in the previous period.

There are three markets, for consumption goods, (interest-yielding) "capital" goods, and labor (power). Normalize the price of the consumption good at $p = 1$, and let (r_t, w_t) denote

the interest and wage rates in period t . All agents are price-takers. Markets are assumed to equilibrate in each period; the process by which they do so is ignored.

Let (L_t, K_t) represent the aggregate stocks of labor power (the “labor force”) and capital at time t . Initially it is assumed that all stocks are fully employed in production; the consequences of allowing for the creation and maintenance of an “industrial reserve army” are considered below. The labor force is assumed to grow at an exogenously given rate g per period. The capital stock is assumed to grow at the rate $sr_t - \mathbf{d}$ in period t , where s is the exogenously given savings rate out of capital income (supposing all labor income is consumed) and \mathbf{d} denotes the exogenously given rate of depreciation of the capital stock.

3. Optimal input use

Assume that each firm projects a demand of x_t for its output, and chooses inputs to minimize the cost of producing that output at going input prices. Thus each firm solves the constrained optimization program

$$\underset{k_t, l_t}{\text{Min}} \quad L = w_t l_t + r_t k_t + \mathbf{I}_t (x_t - k_t^a l_t^{1-a}),$$

which yields the following conditions for an interior optimum (k_t^*, l_t^*) :

$$(3.1) \quad \frac{w_t}{r_t} = \frac{(1-a)k_t^*}{a l_t^*},$$

$$(3.2a) \quad l_t^* = (a w_t / (1-a) r_t)^{-a} x_t,$$

$$(3.2b) \quad k_t^* = (a w_t / (1-a) r_t)^{1-a} x_t.$$

4. Aggregation and macroeconomic consistency

From equations (3.2) and the condition of linearly homogeneous production, aggregate demand for labor and capital inputs is given by

$$(4.1a) \quad L_t^* = (\mathbf{a}w_t/(1-\mathbf{a})r_t)^{-\mathbf{a}} X_t ,$$

$$(4.1b) \quad K_t^* = (\mathbf{a}w_t/(1-\mathbf{a})r_t)^{1-\mathbf{a}} X_t ,$$

where $X_t = Fx_t$. Note that (3.1) also holds for aggregate quantities (K_t^*, L_t^*) .

Input market equilibrium in period t is given by the input prices (r_t^*, w_t^*) which simultaneously equate quantities supplied and demanded in each market, i.e. $K_t^* = K_t$, $L_t^* = L_t$.

. Keynesian-type macroeconomic concerns are sidestepped by assuming that firms' demand expectations are always fulfilled in equilibrium, such that

$$(4.2) \quad w_t^* L_t + r_t^* K_t = X_t , \text{ or}$$

$$(4.2') \quad w_t^* L_t / X_t + r_t^* K_t / X_t = 1.$$

Note this macroeconomic consistency condition does not rule out the existence of an “industrial reserve army”; see section 6 below.

Rearranging (3.2), substituting into (4.2') and simplifying yields

$$(4.3) \quad \left(w_t^* / (1-\mathbf{a}) \right)^{1-\mathbf{a}} \left(r_t^* / \mathbf{a} \right)^{\mathbf{a}} = 1.$$

Hereafter, asterisks are dropped, equilibrium values being assumed to hold throughout.

5. Macrodynamics

Take the total differential of the natural log of (4.3) to yield

$$(5.1) \quad (1 - \mathbf{a}) \frac{dw_t}{w_t} + \mathbf{a} \frac{dr_t}{r_t} = 0,$$

where $\frac{dw_t}{w_t} = dw_t/w_t$, $\frac{dr_t}{r_t} = dr_t/r_t$

Now take the total differential of the aggregate version of (3.1), which produces

$$\frac{r_t dw_t - w_t dr_t}{r_t^2} = \frac{(1 - \mathbf{a})(L_t dK_t - K_t dL_t)}{\mathbf{a} L_t^2}.$$

Recall $dK_t = (sr_t - \mathbf{d})K_t$, $dL_t = gL_t$ by assumption. Substitute these conditions into the previous equation, reapplying (3.1) and simplifying to yield

$$(5.2) \quad \frac{dw_t}{w_t} - \frac{dr_t}{r_t} = sr_t - (\mathbf{d} + g).$$

Substituting (5.2) into (5.1) and simplifying, we have

$$(5.3a) \quad \frac{dw_t}{w_t} = \mathbf{a}[sr_t - (\mathbf{d} + g)],$$

$$(5.3b) \quad \frac{dr_t}{r_t} = (1 - \mathbf{a})[(\mathbf{d} + g) - sr_t].$$

Define the *steady state equilibrium* (r_s, w_s) as the input prices such that $\frac{dw_t}{w_t} = \frac{dr_t}{r_t} = 0$.

Given equations (5.3),

$$(5.4a) \quad r_s = \frac{\mathbf{d} + g}{s},$$

$$(5.4b) \quad w_s = (1 - \mathbf{a}) \left(\frac{\mathbf{a} s}{\mathbf{d} + g} \right)^{\frac{\mathbf{a}}{1-\mathbf{a}}}.$$

These prices can be regarded as the long-run tendencies of the economy for given production technology, population growth rates, and savings rates. If the profit rate falls below r_s , the rate of capital accumulation will slow, leading to a decrease in the wage rate and a corresponding increase in the profit rate until the steady state is re-attained. A parallel story can

be told for profit rates above the steady-state level. Note that the rate of profit in the steady state does not depend on the capital intensity parameter \mathbf{a} .

To anticipate a possible criticism, the result does not depend on the price-inelasticity of input supply, as long as there are levels of supply which are independent of input prices (i.e., “intercept terms”), since price-change terms drop out in the steady state.

6. Consequences of technical change

Capital-using, labor-saving (CU-LS) technical changes can be represented by an exogenously given increase in \mathbf{a} , since this corresponds to an increase in each firm’s capital-labor ratio for given input prices (see (3.1)). Let’s say that a technical change is *viable* if it lowers the average cost of production at given input prices, assuming optimal input use. Thus, a CU-LS innovation $d\mathbf{a} > 0$ is viable if and only if $dAC^*/d\mathbf{a} < 0$, where

$$AC^* = w_t l_t^*/x_t + r_t k_t^*/x_t = \left(w_t/(1-\mathbf{a}) \right)^{1-\mathbf{a}} \left(r_t/\mathbf{a} \right)^{\mathbf{a}}.$$

Thus, viability of a locally CU-LS technical change requires

$$(6.1) \quad dAC^*/d\mathbf{a} = -AC^* \bullet \ln \left(k_t^*/l_t^* \right) < 0,$$

or $k_t^* > l_t^*$. This implies, in equilibrium, $K_t > L_t$, i.e. capital is relatively more plentiful than labor. Let’s say that an innovation is *implemented* if it is universally adopted.

Three sets of comparative statics results are of potential interest: the impact of implementing a (viable) CU-LS innovation on current-period wage and profit rates, on steady-

state wage and profit rates, and on the profit rate given that the wage rate is constrained to be greater than or equal to some level (say the “subsistence” wage rate).

To establish the first set of results, combine (3.1) and (4.2) to yield

$$(6.2a) \quad w_t L_t / (1 - a) = X_t ,$$

$$(6.2b) \quad r_t K_t / a = X_t .$$

In the current period, (K_t, L_t) and thus X_t are invariant. Differentiating (5.2) with respect to a in light of this yields

$$(6.3) \quad dw_t / da = -X_t / L_t < 0, \quad dr_t / da = X_t / K_t > 0,$$

Note however from (5.3) that $da > 0$ changes the magnitudes but not the signs of $(\frac{dw_t}{da}, \frac{dr_t}{da})$.

Turning to the impact of CU-LS technical change on the steady state, it is clear from (5.4a) that the steady-state rate of profit r_s is unaffected by such changes. From (5.4b),

$$(6.4) \quad dw_s / da = \left(\frac{as}{d} + g \right)^{a/(1-a)} \ln(a/r_s) / (1 - a) ,$$

so that $\text{sgn}\left(\frac{dw_s}{da}\right) = \text{sgn}\left(\ln \frac{a}{r_s}\right)$. Thus, for relatively low values of a , implementation of a

CU-LS innovation lowers the steady-state wage rate, and for relatively high values, such innovation raises that wage rate. The lowest possible value of the steady-state wage rate is thus one minus the steady-state rate of profit.

Finally suppose that $w_t = w_{sub}$, the subsistence wage rate, and is constrained to be greater than or equal to it. In this case, using (4.3), we see that

$$(6.5) \quad \frac{dr_t}{da} = r_t \ln(w_{sub}/1-a) / a^2 = r_t \ln(K_t/L_t) / a ,$$

so that implementation of a locally CU-LS technical change raises the current-period equilibrium rate of profit if and only if the innovation is viable. This is simply a restatement of the Okishio theorem in the context of a dynamic system (Roemer (1981)).

7. Conclusions

The model analyzed here produces the following conclusions with respect to Marx's account of capitalist "laws of motion":

- (1) Marx's account of capitalist accumulation in part 1 of Chapter 25 in Volume I is corroborated: if the process of accumulation is such that wage increases cut into the rate of profit, the rate of capital accumulation will tend to slow until a steady state has been restored. This steady state rate of profit is positive even if the "normal rate of profit in the neoclassical sense, i.e. that given by risk aversion and/or impatience, is zero. This result also confirms the dynamic consistency of John Roemer's account of the basis of capitalist profit: relative scarcity of capital is guaranteed by the logic of capitalist accumulation.
- (2) Marx's claim that the process of capitalist accumulation tends to create an "industrial reserve army" is shown to be subject to an important caveat. If optimal steady-state production is relatively labor intensive (i.e., a is less than the steady state rate of profit), then implementation of CU-LS innovations will tend to reduce the steady-state wage rate, potentially creating a steady-state pool of unemployed labor if the aggregate labor supply curve is horizontal at some "subsistence" wage rate $w_{sub} > 1 - r_s$. However, once optimal steady-state

production is relatively capital intensive (i.e., α is greater than the steady-state rate of profit), further implementation of CU-LS innovations increases the steady-state wage rate, which implies full employment of labor power given an “inverse L-shaped” aggregate supply curve for labor power.

(3) Serious doubt is cast on Marx’s theory of “the tendency of the rate of profit to fall”, developed in chapters 13-14 of Volume III. First, the capital intensity of cost-minimizing production has no impact whatsoever on the steady-state rate of profit. Second, if there are no constraints on the wage rate, any locally CU-LS innovation unambiguously raises the current-period rate of profit. If wage rates are constrained to equal some (subsistence) level, then any CU-LS innovation that cost-minimizing capitalists would care to adopt necessarily raises the current-period equilibrium rate of profit upon implementation.

Thus, the only case in which a trend of CU-LS innovation leads in some sense to a fall in the rate of profit is when the capitalist economy is in a steady-state path. Implementing a CU-LS innovation in this case leads eventually to a fall in the rate of profit, other things equal, only because its immediate effect is to raise the profit rate above its steady state level, prompting a subsequent downward adjustment to the original steady-state level. But since this level does not depend on the technical coefficients of production, there is no sense in which such innovations establish a “tendency” for the rate of profit to fall.

If such a tendency is to emerge, it must be the result of some other dynamic than that indicated by Marx. For example, the trend of centralization of capital (i.e. reducing the number and raising the per capita wealth of capitalists) Marx identifies in Chapter 25 of Volume I may result in a tendentially falling (steady-state) profit rate if the marginal propensity to save out of

capital income is increasing in wealth. But this and related issues must be the focus of a separate paper.

References

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